Thermal instability of the giant graviton in a matrix model on a *pp*-wave background

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The thermal instability of the giant graviton is investigated within the Berenstein-Maldacena-Nastase (BMN) matrix model. We calculate the one-loop thermal correction of the quantum fluctuation around the trivial vacuum and giant graviton, respectively. From the exact formula of the free energy we see that at low temperature the giant graviton is unstable and will dissolve into vacuum fluctuations. However, at sufficiently high temperature the trivial vacuum fluctuation will condense to form the giant graviton configuration. The transition temperature of the giant graviton is determined in our calculation.

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I. INTRODUCTION

A matrix model on an 11-dimensional maximally supersymmetric *pp*-wave background has been proposed by Berenstein, Maldacena, and Nastase (BMN) [1,2]. The model provides an interesting example where the string or gauge correspondence can be studied beyond the supergravity approximation [3,4]. In addition to the terms in the Banks-Fischler-Shenker-Sosskinol (BFSS) matrix model [5] the BMN matrix model also contains mass terms and Myers terms [6]. The extra terms can completely remove the moduli space and leave only the discrete set of solutions. Because of the Myers mechanics the Ramond-Ramond (RR) flux (i.e., the extra terms) in the BMN model can expand the brane into its spherical form, i.e., fuzzy spheres [1,2,6]. One of the spheres is the nonsupersymmetric fuzzy sphere while the other one is the supersymmetry [1/2 by Bogomol'nyi-Prasad-Sommerfield (BPS)] fuzzy sphere, which is called a giant graviton [7]. Unlike the similar situation studied by Myers [6] here the all giant graviton solutions have zero energy and are independent of the matrix dimension and representation of SU(2).

In [8] Sugiyama and Yoshida used the background field method to calculate the one-loop quantum correction of the BMN matrix model. They found that no correction is induced around the supersymmetric fuzzy sphere and thus showed the quantum stability of the giant graviton. In this Brief Report we will use their formulations to evaluate the one-loop thermal correction of the BMN matrix model. We evaluate the free energy of the quadratic fluctuations about the trivial vacuum and giant graviton, respectively. We see that while the trivial vacuum and giant graviton solution both have zero energy and do not induce any correction at zero temperature [8], they have different free energies at finite temperature. Our result shows that the giant graviton has higher free energy than the trivial vacuum fluctuation at low temperature. Thus at low temperature the giant graviton will be unstable and dissolve into the trivial vacuum fluctuation. However, at sufficiently high temperature the giant graviton will have less free energy than the trivial vacuum fluctuation. This means that the trivial vacuum fluctuation will condense into giant gravitons, which now become the stable configurations. We also determine the transition temperature of the giant graviton in our calculation.

In Sec. II we briefly review the formulations and results of Sugiyama and Yoshida [8]. In Sec. III we adopt these formulations to evaluate the quantum correction at finite temperature and we draw conclusions in the last section.

II. BMN MATRIX MODEL AND QUANTUM FLUCTUATION

Matrix theory on an 11-dimensional maximally supersymmetric *pp*-wave background is described by the BMN matrix model. The action is given as $S=S_{flat}+S_{\mu}$ where

$$S_{flat}=\int dt\,\mathrm{Tr}\left[\frac{1}{2}D_0X^rD_0X^r+\frac{1}{4}[X^r,X^s]^2+i\Psi^TD_0\Psi-\Psi^T\gamma_r[\Psi,X^r]\right],\qquad r,s=1,2,\ldots,9, \tag{2.1}$$

$$S_{\mu}=\int dt\,\mathrm{Tr}\left[-\frac{1}{2}\left(\frac{\mu}{3}\right)^2X_I^2-\frac{1}{2}\left(\frac{\mu}{6}\right)^2X_{I'}^2-i\frac{\mu}{3}\epsilon_{IJK}X^IX^JX^K-i\frac{\mu}{4}\Psi^T\gamma_{123}\Psi\right],\qquad I=1,2,3,\quad I'=4,5,\ldots,9, \tag{2.2}$$

where the covariant derivative is defined by $D_0X^r\equiv\partial_tX^r-i[A,X^r]$ and we have rescaled the gauge field A and parameters t and μ as $t\to t/R$, $A\to RA$, and $\mu\to R\mu$ in which R is the radius of circle compactification along x^- [1].

To proceed, we first decompose X^r and Ψ into the backgrounds B^I $(=\alpha J^I)$,F and fluctuations Y^r,ψ as

$$X^r=B^r+Y^r,\qquad \Psi=F+\psi, \tag{2.3}$$

where we take $F=0$ since the fermionic background is not considered in this paper. When $\alpha=\mu/3$ the background is the giant graviton while the case of $\alpha=0$ is the trivial vacuum solution. Note that the giant graviton is a solution of Eq. (2.1) and has zero energy like the trivial solution [1,2]. Next,

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TABLE I. The spectrum of fluctuations in a giant graviton.

Field	Bosonic fluctuations Y					Gauge fields A		Ghost fields C		Fermions ψ		
Degeneracy	4	3	5	6	18	1	3	1	3	16	32	16
Mass	$\frac{\mu}{3}$	$\frac{\sqrt{2}\mu}{3}$	$\frac{2\mu}{3}$	$\frac{\mu}{6}$	$\frac{\mu}{2}$	0	$\frac{\sqrt{2}\mu}{3}$	0	$\frac{\sqrt{2}\mu}{3}$	$\frac{\mu}{4}$	$\frac{7\mu}{12}$	$\frac{-5\mu}{12}$

we adopt the background field gauge, and the gauge-fixing terms and Faddeev-Popov ghost terms are

$$S_{GF+FP}=-\frac{1}{2}\int dt\, \mathrm{Tr}\{(D_0^{bg}A)^2+i\bar{C}\partial_t D_0 C + \bar{C}[B^r,[X^r,C]]\}, \tag{2.4}$$

where $D_0^{bg}A$ is defined by $D_0^{bg}A=\partial_t A+i[B^r,X^r]$. The advantage of this gauge choice is that the second order actions with respect to the fluctuations are simplified [8].

For the case of 2×2 matrices we can expand the fluctuations and gauge fields as

$$Y^r=\frac{1}{\sqrt{2}}Y_0^r\mathbf{1}_2+\sqrt{2}Y_1^rJ^1+\sqrt{2}Y_2^rJ^2+\sqrt{2}Y_3^rJ^3,$$

$$\psi=\frac{1}{\sqrt{2}}\psi_0\mathbf{1}_2+\sqrt{2}\psi_1J^1+\sqrt{2}\psi_2J^2+\sqrt{2}\psi_3J^3,$$

$$A=\frac{1}{\sqrt{2}}A_0\mathbf{1}_2+\sqrt{2}A_1J^1+\sqrt{2}A_2J^2+\sqrt{2}A_3J^3, \tag{2.5}$$

where $J^I\equiv\sigma^I/2(I=1,2,3)$ and the σ^I's are Pauli matrices. Substituting these terms into Eqs. (2.1) and (2.2) and diagonaling we see that the quadrature fluctuations about the giant graviton have the content of fields and spectra given in Table I.

After integrating the quadratic fluctuations the one-loop effective action W has four contributions. The contribution of the fluctuations Y, gauge field A, ghost C ($\bar{C}$), and fermion parts F are given, respectively, by

$$W_Y=-\ln\left(\mathrm{Det}\left[-\partial_\tau^2+\frac{\mu^2}{9}\right]^{-4/2}\mathrm{Det}\left[-\partial_\tau^2+\frac{2}{9}\mu^2\right]^{-3/2}\times\mathrm{Det}\left[-\partial_\tau^2+\frac{4}{9}\mu^2\right]^{-5/2}\mathrm{Det}\left[-\partial_\tau^2+\frac{\mu^2}{36}\right]^{-6/2}\times\mathrm{Det}\left[-\partial_\tau^2+\frac{\mu^2}{4}\right]^{-18/2}\right), \tag{2.6}$$

$$W_A=-\ln\mathrm{Det}\left[-\partial_\tau^2+\frac{2}{9}\mu^2\right]^{-3/2},$$

$$W_{gh}=-\ln\mathrm{Det}\left[-\partial_\tau^2+\frac{2}{9}\mu^2\right]^{3}, \tag{2.7}$$

$$W_F=-\ln\left(\mathrm{Pf}\left[i\partial_\tau+\frac{\mu}{4}\gamma_{123}\right]\mathrm{Pf}\left[i\partial_\tau+\frac{7}{12}\mu\gamma_{123}\right]^2\times\mathrm{Pf}\left[i\partial_\tau-\frac{5}{12}\mu\gamma_{123}\right]\right), \tag{2.8}$$

where $[\mathrm{Pf}(B)]^2=\mathrm{Det}(B)$. Finally, using the property derived in [8],

$$\ln\mathrm{Det}[-\partial_\tau^2+M^2]=L\int\frac{dk_0}{2\pi}\ln(k_0^2+M^2)=L(|M|+E_\infty), \tag{2.9}$$

$$\ln\mathrm{Pf}[i\partial_\tau+M\gamma_{123}]=4L\int\frac{dk_0}{2\pi}\ln(k_0^2+M^2)=4L(|M|+E_\infty), \tag{2.10}$$

where L is the length of the temporal direction and

$$E_\infty=\int\frac{dk}{2\pi}\ln k^2, \tag{2.11}$$

the net one-loop contribution is exactly canceled as expected from the supersymmetry. This is the result of [8].

To proceed we need to know the quadrature fluctuations about the trivial vacuum. Following the above procedure we have the fields and spectra given in Table II. From these spectra the net one-loop corrections in the trivial vacuum can also be evaluated in the same way. It is easy to see that they are exactly canceled as expected from the supersymmetry.

III. ONE-LOOP THERMAL CORRECTION AND FREE ENERGY

At finite temperature T $(=1/\beta)$ the integration of k_0 in Eqs. (2.9) and (2.10) is replaced by a summation over $2n\pi/\beta$. The summation can be performed by the following relations.

For the case of the boson n is the natural integral (NI) and we have the relation

TABLE II. The spectrum of fluctuations in the trivial vacuum.

Fields	Bosonic fluctuations Y		Gauge fields A	Ghost fields C	Fermions ψ
Degeneracy	12	24	4	4	64
Mass	$\frac{\mu}{3}$	$\frac{\mu}{6}$	0	0	$\frac{\mu}{4}$

$$\sum_{n\in \mathrm{NI}} \ln[(2\pi n)^2+(\beta M)^2] = \sum_{n\in \mathrm{NI}} \int_1^{(\beta M)^2} \frac{ds^2}{(2\pi n)^2+s^2} + \sum_{n\in \mathrm{NI}} \ln[1+(2\pi n)^2]$$
$$= \int_1^{(\beta M)^2} \frac{ds^2}{2s}\left[1+\frac{2}{e^s-1}\right] + \sum_{n\in \mathrm{NI}} \ln[1+(2\pi n)^2]$$
$$= (\beta M-1)+2[\ln(1-e^{-\beta M})-\ln(1-e^{-1})] + \sum_{n\in \mathrm{NI}} \ln[1+(2\pi n)^2]. \tag{3.1}$$

In the same way, for the case of the fermion n is the half integral (HI) and we have the relation

$$\sum_{n\in \mathrm{HI}} \ln[(2\pi n)^2+(\beta M)^2] = \sum_{n\in \mathrm{HI}} \int_1^{(\beta M)^2} \frac{ds^2}{(2\pi n)^2+s^2} + \sum_{n\in \mathrm{HI}} \ln[1+(2\pi n)^2]$$
$$= \int_1^{(\beta M)^2} \frac{ds^2}{2s}\left[1-\frac{2}{e^s+1}\right] + \sum_{n\in \mathrm{HI}} \ln[1+(2\pi n)^2]$$
$$= (\beta M-1)+2[\ln(1+e^{-\beta M})-\ln(1+e^{-1})] + \sum_{n\in \mathrm{HI}} \ln[1+(2\pi n)^2]. \tag{3.2}$$

Then, using the spectrum in Tables I and II we can easily calculate the partition function and thus the free energy. The exact formula for the partition function is

$$\ln Z = \sum_{i\in Y,A}\left(\frac{-N_i}{2}\right)\Bigg[(\beta m_i-1)+2[\ln(1-e^{-\beta m_i}) - \ln(1-e^{-1})] + \sum_{n\in \mathrm{NI}} \ln[1+(2\pi n)^2]\Bigg] + \sum_{i\in C,\psi}\left(N_i\delta_{i,C}+\frac{N_i\delta_{i,\psi}}{4}\right) \times\Bigg[(\beta m_i-1)+2[\ln(1+e^{-\beta m_i})-\ln(1+e^{-1})] + \sum_{n\in \mathrm{HI}} \ln[1+(2\pi n)^2]\Bigg], \tag{3.3}$$

in which N_i and m_i are the degeneracy and mass of the fluctuation field of type i. After the numerical evaluation, we see that the free energy of fluctuation from the trivial vacuum is lower than that from the giant gravity system at low temperature. However, when the temperature is larger than the critical temperature T_c,

$$T_c \approx 385\mu, \tag{3.4}$$

then the free energy of the giant graviton becomes less than that of the trivial vacuum. The analytic relations can be obtained in the high and low temperature cases. For example, in the high temperature limit we have the simple relation

$$F_{giantgraviton}-F_{trivalvacuum} \approx -3T\ln(T/\mu), \quad T \gg \mu, \tag{3.5}$$

while in the low temperature limit we have

$$F_{giantgraviton}-F_{trivalvacuum} \approx 18Te^{-\mu/6T}, \quad T \ll \mu. \tag{3.6}$$

We thus conclude that at high temperature the vacuum fluctuation will condense into the configuration of the giant graviton. On the other hand, at low temperature the giant graviton will become unstable and dissolve into vacuum fluctuations. The property is shown in Fig. 1.

IV. CONCLUSION

In this paper we use the formulations of Sugiyama and Yoshida [8] to evaluate the one-loop thermal correction in the BMN matrix model. We evaluate the free energy of the quadratic fluctuations about the trivial vacuum and giant graviton, respectively. We see that while the trivial vacuum and giant graviton solutions both have zero energy and do not induce any correction at zero temperature [8], the trivial vacuum will have less free energy than the giant graviton

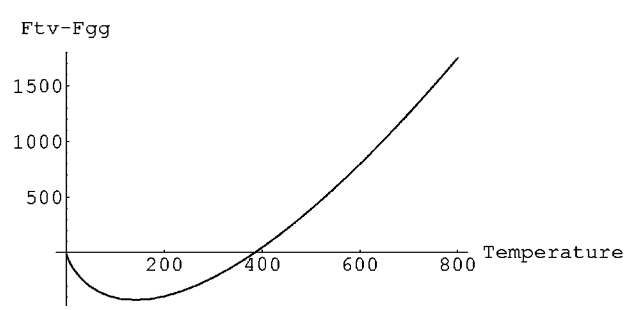

FIG. 1. Temperature dependence of the difference between the trival-vacuum free energy F_{tv} and giant-graviton free energy F_{gg}. The scale is in units of $\mu=1$.

when the temperature is increased. Therefore at low temperature the giant graviton will be unstable and dissolve into trivial vacuum fluctuations. However, at sufficiently high temperature the giant graviton will have less free energy than the trivial vacuum fluctuation. This means that the trivial vacuum fluctuation will condense into giant gravitons which now become the stable configurations at high temperature. The phenomenon of a physical configuration that becomes stable at high temperature can also be found in other systems. The simplest one may be the condensation of the topological vortex at high temperature in the two-dimensional *XY* statistical model [9].

It is hoped that the phase transition of the giant graviton that we find may be relevant to cosmological evolution in which the temperature is cooling down during the expansion of the universe.

Finally, we want to mention that Bak [10] dealt with the giant graviton and its elliptic deformation with eight-dimensional supersymmetry. The computation of the free energy of these elliptic configurations and investigation of their thermal stability would be quite interesting. It remains to be studied.

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